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A transport-distance approach to scaling erosion rates: I. Background and model development

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Abstract

The process basis of existing soil-erosion models is shown to be ill-founded. The existing literature builds directly or indirectly on Bennett's (1974) paper, which provided a blueprint for integrated catchment-scale erosion modelling. Whereas Bennett recognized the inherent assumptions of the approach suggested, subsequent readings of the paper have led to a less critical approach. Most notably, the assumption that sediment movement could be approximated by a continuity equation that related to transport in suspension has produced a series of submodels that assume that all movement occurs in suspension. For commonly occurring conditions on hillslopes, this case is demonstrably untrue both on theoretical grounds and from empirical observations. Elsewhere in the catchment system, it is only partially true, and the extent to which the assumption is reasonable varies both spatially and temporally. A second ground-breaking paper - that of Foster and Meyer (1972) - was responsible for subsequent uncritical application of a first-order approximation to deposition based on steady-state analysis and again a weak empirical basis. We describe in this paper an alternative model (MAHLERAN - Model for Assessing Hillslope-Landscape Erosion, Runoff And Nutrients) based upon particle-travel distance that overcomes existing limitations by incorporating parameterizations of the different detachment and transport mechanisms that occur in water erosion in hillslopes and small catchments. In the second paper in the series, we consider the sensitivity and general behaviour of MAHLERAN, and test it in relation to data from a large rainfall-simulation experiment. The third paper of the sequence evaluates the model using data from plots of different sizes in monitored rainfall events. From this evaluation, we consider the scaling characteristics of the current form of MAHLERAN and suggest that integrated modelling, laboratory and field approaches are required in order to advance the state of the art in soil-erosion modelling. Copyright © 2008 John Wiley & Sons, Ltd.

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Introduction

Attempts to develop models of soil erosion by water date back over half a century (e.g. Zingg, 1940). Such models are important for understanding the impact of agricultural practices and land-use change in the short term, reservoir sedimentation in the medium term and, in the longer term, the evolution of the landscape. Since the 1980s, the focus has moved away from regression-based empirical methods towards models that have a more explicit representation of processes. Despite the increasing complicatedness of models, there appears to have been little overall improvement in their predictive capability over the short term, even at small spatial scales (see, e.g., Favis-Mortlock, 1998; Jetten *et al.*, 1999; Tiwari *et al.*, 2000). Furthermore, the results of existing approaches remain incompatible with rates of long-term landscape evolution and with estimations over large spatial scales (Parsons and Abrahams, 1992, p. x;

Wainwright *et al.*, 2001; Parsons *et al.*, 2004, 2006b). We have argued elsewhere (Parsons *et al.*, 2004) that these problems arise from a fundamental misconception, and thus misrepresentation, of the component processes that make up soil erosion. The aim of this paper is, therefore, to review the conceptual basis of existing models and to propose an alternative model that addresses the problems inherent in existing approaches.

Limitations of Existing Soil-Erosion Models

Bennett (1974) is frequently cited (see, e.g., Morgan *et al.*, 1998; Woolhiser *et al.*, 1990; Kirkby, 1978; Foster, 1990; Lane *et al.*, 1988; Mitas and Mitasova, 1998) as providing the rationale underlying the development of most current, process-based, soil-erosion models. In his paper, Bennett argues that 'three differential equations form the mathematical basis for model[1]ing both phases of sediment yield. They are the equations that describe the movement of *suspended sediment particles* in a one-dimensional, infinitely wide, free surface flow' (1974, p. 486; emphasis added). The first two equations describe the continuity of mass and momentum in the flow of water. The third describes the conservation of mass for sediment transport, given as

$$\frac{\partial(hC)}{\partial t} + (1 - \lambda)\frac{\partial y}{\partial t} + \frac{\partial(hu_pC)}{\partial x} = \frac{\partial}{\partial x}h\eta_p\frac{\partial C}{\partial x}$$
(1)

where h is the depth of water flow [m]

C is the sediment concentration in the flow $[m^3 m^{-3}]$

t is time [s]

 λ is the porosity of the sediment at the surface [m³ m⁻³]

y is the elevation of the surface [m]

 $u_{\rm p}$ is the velocity of the sediment in movement [m s⁻¹]

x is the distance along the surface [m]

 $\eta_{\rm p}$ is the sediment-particle mass-transfer coefficient [m² s⁻¹].

The two phases to which Bennett refers are the upland (hillslope) and lowland (channel) phases of movement. In applying this approach, two simplifications are usually made. First, the velocity of the particles in movement is assumed to be identical to the velocity of the surrounding flow (although crucially Bennett (1974) notes that 'it cannot necessarily be assumed that the transport velocity of the suspended substance is the same as the flow velocity'). Secondly, the term on the right-hand side, which describes the dispersion of the suspended sediment in the flow, is assumed to be negligible compared to the other terms, and is thus set to zero. To evaluate the second term in Equation (1), which describes the change in elevation of the surface, or in other words the net erosion (i.e. total erosion less the deposition), Bennett (1974) suggested following the approach of Foster and Meyer (1972), which considers net erosion as a function of raindrop and flow processes, so that

$$D_{\rm F} + R_{\rm DT} = (1 - \lambda)\gamma \rho_{\rm s} \frac{\partial y}{\partial t}$$
⁽²⁾

where $D_{\rm F}$ is the net flow-detachment rate [kg m⁻² s⁻¹]

 $R_{\rm DT}$ is the net raindrop-detachment rate [kg m⁻² s⁻¹]

 γ is the unit weight of water [kg m⁻³]

 $\rho_{\rm s}$ is the sediment-particle density [kg m⁻³].

For the channel phase of sediment transport, other approaches are suggested as feasible, but a general approach is cast in these same terms.

Combining these assumptions, and simplifying further to assume that most parameterizations of net erosion rates include the effect of the change in volume relating to the porosity of the surface, produces the form of equation that is now generally presented:

$$\frac{\partial(hC)}{\partial t} + \frac{\partial(qC)}{\partial x} - e = 0$$
(3a)

or

$$\frac{\partial(AC)}{\partial t} + \frac{\partial(QC)}{\partial x} - E = 0$$
(3b)

where q is the unit discharge of water $[m^2 s^{-1}]$ (= hu)

e is the unit net erosion rate $[m s^{-1}] (= \frac{D_F + R_{DT}}{(1 - \lambda)\gamma \rho \frac{\partial y}{\partial t}}, \text{ or parameterized directly in terms of } \frac{\partial y}{\partial t})$

A is wetted surface area of the flow $[m^2]$ (= hw; where w is the width of flow in m)

Q is water discharge $[m^3 s^{-1}]$ (= *huw*)

E is the areal erosion rate $[m^2 s^{-1}]$ (= *ew*).

Equations (3a) and (3b) differ simply because they simulate unit and volumetric conditions, respectively. Some versions also include lateral inflow terms to the right-hand side of these equations. The equations are either derived directly from Bennett (1974) (e.g. Govindaraju and Kavvas, 1991; Morgan *et al.*, 1998; Woolhiser *et al.*, 1990) or indirectly (e.g. Rose *et al.*, 1983; Rose, 1985; Jetten, 2003; Yu, 2003) by reference to the other aforementioned literature that cites the approach.

A significant issue arises from the definition of erosion rates in these terms. The approach assumes (explicitly in the case of Bennett, but often implicitly in subsequent works) that sediment is transported through the system in suspension. However, in the case of hillslope erosion, at least, transport in suspension rarely occurs. Empirical observations suggest that erosion in interrill flow is dominated by rolling or sliding along the surface, or in short steps akin to the movement of bedload (Wainwright and Thornes, 1991; Parsons et al., 1993, 1998; Rejman et al., 1999), and that, in rills, gullies and channels, bedload can make up a significant proportion of the sediment in transport, depending on local conditions (Guy et al., 1966; Parker, 2007, Figure 3.29; Dietrich and Whiting, 1989; Torri et al., 1990). Taking the theoretical definition of Van Rijn (1984), it can be seen that the theory also supports these empirical observations (Figure 1). On most hillslopes, transport in suspension will rarely occur (for all but the finest particles, which generally form aggregates that will not cross the threshold for movement, unless these aggregates break down), and the extent to which it occurs elsewhere will vary differentially. If consideration is made of the fact that most surfaces are rougher than the smooth conditions under which Van Rijn's definition is made, it should be the case that even less suspension is developed, not least because of the case where the surface irregularities provide traps for movement (see, e.g., Wainwright and Thornes, 1991, although Darboux and Huang, 2005, note that the relative importance of this process will depend on a variety of factors including soil and roughness conditions and their interaction). Furthermore, the turbulent flow régime is rarely established by definition in unconcentrated overland flows. An immediate implication of these considerations is that the use of the flow discharge to rate the movement of sediment (the second term in Equation (3a) or (3b)) will lead to significant overestimates of sediment transport on hillslopes, and variable overestimates elsewhere. In consequence, the process basis of most commonly used, process-based erosion models is incorrect.

A further implication of this argument is evident in the calculation of the net erosion terms, especially in relation to the flow-detachment component. Foster and Meyer (1972) argued on theoretical grounds that if steady-state conditions were considered, then

$$\frac{D_{\rm F}}{D_{\rm C}} + \frac{G_{\rm F}}{T_{\rm C}} = 1 \tag{4}$$

where $D_{\rm C}$ is the detachment capacity of the flow [kg m⁻² s⁻¹]

 $G_{\rm F}$ is the sediment load in the flow [kg m⁻¹ s⁻¹]

 $T_{\rm C}$ is the transport capacity of the flow [kg m⁻¹ s⁻¹].

Based on descriptions of laboratory experiments, channel degradation below dams and deposition at the toe of slopes (assuming an apparent equivalence of detachment and deposition rates), they stated that 'the rate of detachment or deposition by flow was concluded to be a function of the difference between the actual sediment load and the capacity of the flow to transport sediment' (Foster and Meyer, 1972, pp. 12–13), or

$$D_{\rm F} = k(T_{\rm C} - G_{\rm F}) \tag{5}$$

where k is a rate coefficient $[m^{-1}]$ defined by putting $D_{\rm C} = kT_{\rm C}$.

The assumption that the rate coefficient can be defined thus was based on empirical (but unpublished other than as a conference abstract) observations that $T_{\rm C} = C_t \tau^{1.5}$, at large shear stresses (where C_t is a coefficient and τ is the flow



Figure I. Thresholds for transport in suspension for different particle sizes according to the criterion of Van Rijn (1984) (suspension will occur for conditions above the horizontal lines) for a range of slope angles and flow depths (diagonal lines represent calculated values of shear velocity, u_{*} , for given flow depths and slopes).

shear stress [N]), and an argument by analogy that $D_{\rm C} = C_{\rm d} \tau^{1.5}$ (where $C_{\rm d}$ is a coefficient; and thus $k = C_{\rm d}/C_{\rm t}$), again supported by unpublished observations.

Bennett (1974, p. 491) stated that the understanding of this approach was incomplete and that 'there is a need for investigation of the influence of unsteadiness and of flow noncomformities on sediment transport characteristics. This would clarify whether or not it is proper to define the response of the sediment load to changes in transport and detachment capacity by a first-order reaction law [such as that given by Equation (5)]' (emphasis added). However, the approach has been used widely in the literature with little subsequent testing or questioning. Foster and Meyer (1972, pp. 12–13) produced empirical support for their approach by stating that 'Meyer and Monke [1965] and Willis [1971] noted that the rate of erosion (detachment) at the head of a noncohesive bed where flow was introduced depended on the amount of sediment in the added flow'. In the case of Willis (1971), the experiments in question were carried out only with added sediment concentrations of 20 000 ppm followed by 0 ppm. Although he noted that the transport rate decreased asymptotically with time, this observation in itself is not necessarily consistent with the postulated equation of Foster and Meyer, and would only apply to a net depositional régime. Although Zheng et al. (2000) remarked on the limited empirical basis for this approach, their experiments were also based on steady-state assumptions, despite the fact that their experiments resulted in the development of concentrated flows and surface evolution that was clearly not steady state (see also Huang et al., 1999, and the similar approaches of Beuselinck et al., 1999, 2002). Furthermore, Foster and Meyer say that their approach follows that of Einstein (1942), which is, again, based on the study of suspended sediment. Although there have subsequently been some limited tests of the Foster-Meyer approach, they have still only focussed on relatively narrow sets of conditions (Rice and Wilson, 1990; Cochrane and Flanagan, 1996), or employ experimental designs that are based on inappropriate representations of the

sediment-transport process (Merten *et al.*, 2001). There is thus no strong support for assuming that an equation derived for steady-state conditions should be a good basis for a model of unsteady erosion.

Approaches based on the assumption that a specific flow possesses a specific capacity to transport sediment may also be called into question. In particular, the approach ignores the fact that at increasing sediment concentrations the nature of the flow will change, producing first hyperconcentrated and then ultimately débris flows. The use of transport-capacity approaches thus reduces the ability of erosion models to simulate the full range of the erosion process, and thus conditions under which erosion events may be particularly damaging. However, the range of sediment concentrations produced in most applications of erosion models is insufficient to require the development of this broader approach, especially as it would require further modifications to the flow model. Notwithstanding these limitations, it remains the case that current transport-capacity approaches are poor representations of the processes in operation. There are three further issues about transport capacity that need to be considered. First, the Yalin (1972) equation assumes that sediment motion begins when the lift force of the flow exceeds a critical lift force. The particle is transported downstream until the particle weight forces it out of the flow and back to the bed. The number of particles in motion at any one time is a linear function of an excess shear parameter. Ferro (1998, p. 1895) states that 'particle detachment stops when the sediment load G is equal to T_c ', which is not equivalent. These conditions are furthermore not met for the case where unconcentrated erosion is being driven by raindrop detachment. Secondly, the equation for calculating the transport capacity includes D, the size of the sediment. Therefore, T_c is size specific and is inversely proportional to D. The simplification noted above that $T_c = C_t \tau^{1.5}$ is only appropriate when τ is very large compared to τ_c , which does not occur in unconcentrated overland flows. Even discounting this issue, it is questionable whether the equation is useful when for some of the available sediment, $\tau < \tau_c$, and even for that which has $\tau_c < \tau$ its availability for entrainment will be affected by that for which the converse is the case (see also the debate in the fluvial literature regarding selective versus equal mobility: Powell, 1998). Thirdly, the Yalin equation is a bedload equation and so was only ever intended to work for bedload conditions, while its uncritical application to other transport conditions again means that models contain conceptually flawed and empirically unjustified components.

The impossibility of theoretically deriving or measuring appropriate parameters means that Equation (5) is generally calibrated for application in models. Given this need for calibration and the lack of clear physical meaning of the rate coefficient k (Equation [5]), it is unlikely that the equation of Foster and Meyer (1972) is a good representation of the erosion process, and thus there is a second major flaw in the process basis of most existing erosion models. This flaw is exacerbated when one considers that most models attempt to reproduce the dynamics of erosion through time, yet are based on the definition of steady-state conditions. Examples of this problem can be seen both with WEPP (Laflen *et al.*, 1991; Risse *et al.*, 1995; Zhang *et al.*, 1996), KINEROS/KINEROS2 (Smith *et al.*, 1995; Woolhiser *et al.*, 1990; Goodrich *et al.*, 2002) and EUROSEM (Morgan *et al.*, 1998). The reliability of the relationship is likely to be called into question even more during dynamically changing conditions, where the assumption that the threshold shear stress for movement has a negligible effect (and thus the basis of the definition of the rate coefficient in Equation (5), which depends on the assumption $\tau > \tau_c$) is unlikely to be met, especially in unconcentrated flow conditions on hillslopes (where $\tau << \tau_c$ or at best $\tau \approx \tau_c$). As noted above, the limitations of this approach had been recognized by Bennett (1974, p. 491). Given the difficulties with the approach, it is appropriate, over 30 years on, to develop a conceptually more robust method.

The net erosion term of erosion models can, alternatively, be approached by considering the deposition of sediment explicitly. Such models (e.g. Rose, 1985) use the settling velocity as calculated from Stokes's law assuming settling from a static body of water by sediment in suspension. Again, the fallacy that sediment transport occurs in suspension seems to have been caused by the form of sediment continuity, and there is a misrepresentation of the processes at work. Little attention has been paid to deposition under other forms of transport. Subsequent failures of this alternative approach result in various forms of direct calibration (e.g., Lisle *et al.*, 1998, introduced a factor relating to sediment concentrations close to the bed) or subsequent overall calibration of the model. A hybrid approach has been taken in models such as EUROSEM (Morgan *et al.*, 1998) and LISEM (Jetten, 2003), where the rate coefficient *k* is assumed to equal the settling velocity multiplied by a 'flow detachment efficiency coefficient' (Morgan *et al.*, 1998, p. 536). This hybrid approach has been required because the settling-velocity approach leads to significant underestimations of local deposition compared with sediment that is effectively moving as bedload.

The paper of Bennett (1974) was insightful in many ways, not least in attempting to provide an integrated framework for the estimation of erosion and sediment transport through a catchment (cf. Kirkby, 1992, p. 159). The framework should provide a basis for scaling erosion estimates (e.g. the ideal underlying WEPP that the approach can be applied from plot, through hillslope, to small catchment scale: Laften *et al.*, 1991), but fails to do so because it misrepresents the processes at work. Furthermore, it is unfortunate that subsequent authors have failed to be as cautious as Bennett in realizing the limitations of the approaches presented and the need for further theoretical and empirical work on key issues. Rather, the papers of Bennett (1974) and of Foster and Meyer (1972) have gained almost mythological status, producing a suite of erosion models that claim to be process based, but which, in fact, have little demonstrated basis in reality.

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A new approach

The weaknesses of existing models of soil erosion demonstrate that a new approach needs to be developed. Such an approach needs to address three issues. First, it should be based on observed forms of sediment transport and their characteristics. Secondly, the nature of the depositional process under these different forms needs to be accounted for. Thirdly, the approach should be capable of producing improved estimates of erosion from hillslope to catchment and, ultimately, landscape scales.

Parsons *et al.* (2004) argued on theoretical grounds that appropriate spatial scaling of erosion estimates could be made employing an approach based on the description of the transport process using the travel distances of particles (see also the discussion by Kirkby, 1991, 1992). Given the current lack of process description of deposition in unconcentrated and concentrated flows, it is argued here that the use of travel distances is a valuable empirical approach to the definition of the deposition process under a range of conditions. As the physical process of deposition is also likely to depend on a range of variables that are not easily measurable or parameterizable (e.g. micro-scale variability in the bed surface and its interaction with flow conditions, especially turbulence), it is argued that this definition is the best *process-based* description available that lends itself to numerical modelling. Although analytical approaches to transport-distance-based models (Wainwright *et al.*, 2001; Parsons *et al.*, 2004) have produced useful predictions that have been successfully tested using field experiments (Parsons *et al.*, 2006a), the utility of analytical approaches is limited to restricted, relatively static conditions. A more flexible approach can be achieved through numerical modelling. Below, we present such an approach.

Model Structure

Given the conceptual difficulties raised by the application of a sediment-continuity equation based on sediment concentration, it is useful to consider an approach that can treat sediment transport more generally. In other words, an approach is needed that contains no implicit or explicit *assumption* of transport in suspension but nevertheless can represent those conditions where suspension does occur. A form of the continuity equation for sediment transport more general than Equation (3) can be given in terms of sediment discharge per unit width as

$$\frac{\partial h_s}{\partial t} + \frac{\partial q_s}{\partial x} - \varepsilon + d = 0$$
(6a)

or in volumetric terms as

$$\frac{\partial(wh_{\rm s})}{\partial t} + \frac{\partial Q_{\rm s}}{\partial x} - w(\varepsilon + d) = 0 \tag{6b}$$

where h_s is the equivalent depth of sediment in transport [m]

 q_s is the unit discharge of sediment [m² s⁻¹]

 ε is the rate of erosion of the surface [m s⁻¹]

- d is the rate of deposition $[m s^{-1}]$
- $Q_{\rm s}$ is the discharge of sediment [m³ s⁻¹].

Equation (6) (a or b) can be parameterized appropriately to account for any form of sediment transport on hillslopes and in channels. In this paper, parameterizations will be given for splash, interrill-overland-flow erosion, concentratedflow erosion and transport by suspension using the travel-distance description of particle movement. To enable selective transport to be simulated, the sediment needs to be divided into size classes, the movement of which is simulated separately. In general terms, Equation (6a) thus becomes

$$\frac{\partial h_{\mathrm{s},\varphi}}{\partial t} + \frac{\partial q_{\mathrm{s},\varphi}}{\partial x} - \varepsilon_{\varphi} + d_{\varphi} = 0 \tag{6c}$$

where the subscript φ is an index relating to a specific size class of sediment. The current version of the model uses six size classes as elaborated by Wainwright *et al.* (1995, 1999a). An equivalent volumetric form is obtained by multiplying Equation (6c) through by w.

One problem with approaches based on transport distance (e.g. the stochastic models of Einstein, 1950; Hubbell and Sayre, 1964; Yang and Sayre, 1971) is that they scale poorly with time, especially when considering highly dynamic

conditions requiring short time steps. The reason for this problem is the general assumption that particles remain at rest for a finite time and then jump instantaneously to their new positions. As model time steps decrease, this assumption breaks down and the approach can start to overestimate sediment transport significantly, depending on the relative values of dt and u_p (the velocity of sediment movement). The approach taken here is to approximate u_p with a value of virtual transport rate or virtual velocity, and assume that it can be parameterized directly – either from empirical observations of distance moved by a sediment particle per unit time, or by theoretical calculation by differentiation of transport distance with respect to time. We thus define

$$q_{s,\varphi} = h_{s,\varphi} v_{p,\varphi} \tag{7a}$$

and

$$Q_{s,\varphi} = h_{s,\varphi} v_{p,\varphi} w \tag{7b}$$

where $v_{p,\phi}$ is the virtual velocity of transport of a sediment particle of size class ϕ [m s⁻¹].

This definition of sediment discharge using particle virtual velocity is an important difference between this version of the sediment-continuity equation and the similar form presented by Foster (1982). The use of virtual velocity produces an implicit spatial scaling of erosion with a sound process basis. Foster's use of *flow* velocity to calculate sediment discharge, on the other hand, does not (and ultimately derives from the issue of confusion of transport mechanisms and the simplification of Bennett as noted above).

Changes in erosion rate through time are a function of variations in precipitation and flow. Given knowledge of rainfall and an appropriate hydrologic-hydraulic model, changes in these rates can be estimated. The model used for present purposes is described in detail elsewhere (Scoging, 1992; Parsons *et al.*, 1997; Wainwright and Parsons, 2002) but briefly consists of an infiltration component (using either a simplified Green-Ampt or a Smith-Parlange [1978] approach), which allows the determination of runoff by Hortonian or saturation overland flow mechanisms. Runon infiltration is also simulated. Runoff is routed using the steepest descent method on a finite difference grid, in which flow is calculated using a kinematic wave approximation to the Saint-Venant equations. Flow velocity is calculated dynamically as a function of flow depth and surface roughness (represented by the Darcy–Weisbach friction factor, *ff*). Infiltration and roughness parameters are fully distributed spatially.

Three detachment conditions are simulated. First, at times or in locations where no flow is present (such as at the start of a rainfall event before ponding), erosion is calculated as a function of raindrop detachment. For present purposes, this detachment is parameterized using the results of Quansah (1981). While it is appreciated that this is a relatively limited empirical basis, it is the only approach to our knowledge that allows different rates of detachment as a function of particle size, rainfall kinetic energy and surface slope to be calculated (see Wainwright *et al.*, 1995):

$$\varepsilon_{\mathbf{r},\varphi} = a_{\varphi} \mathbf{K} \mathbf{E}^{b_{\varphi}}, \qquad S = 0 \tag{8a}$$

$$\varepsilon_{r,\varphi} = a_{\varphi} \mathrm{KE}^{b_{\varphi}} S^{c_{\varphi}}, \qquad S > 0 \tag{8b}$$

where $\varepsilon_{r,\varphi}$ is raindrop detachment rate for particle size class φ [m s⁻¹]

KE is rainfall kinetic energy per unit area per unit precipitation [J m⁻² mm⁻¹]

S is surface slope $[m m^{-1}]$

 $a_{\varphi}, b_{\varphi}, c_{\varphi}$ are empirical parameters that are a function of particle size and density.

It is likely that parameters a_{φ} , b_{φ} , c_{φ} will vary dynamically according to changes in soil cohesion (Al Durrah and Bradford, 1982; Nearing and Bradford, 1985; Bradford *et al.*, 1987b), surface sealing (Bradford *et al.*, 1986, 1987a), aggregate stability (Farres, 1987), organic matter content (Tisdale and Oades, 1982), soil compaction (Drewry *et al.*, 2004) and particle arrangement (Torri, 1987), but at present there are insufficient empirical data for the parameterization of these dynamics, so that they are not included in the current model (see further discussion by Wainwright *et al.*, in press b). Where vegetation is present, the kinetic energy is scaled to account for changes in energy relating to interception losses, throughfall and leafdrip. Secondly, unconcentrated overland flow erosion (interrill erosion) is simulated using the raindrop-detachment rate as described above, but modifying the rate to account for the protective effects of the surface-water layer. The exponential model of Torri *et al.* (1987) is used, but accounting for the parameterization issue noted by Parsons *et al.* (2004):

$$\varepsilon_{\mathbf{u},\boldsymbol{\varphi}} = \varepsilon_{r,\boldsymbol{\varphi}} e^{-\beta_{\boldsymbol{\varphi}} h} \tag{9}$$

where $\mathcal{E}_{u,\phi}$ is unconcentrated flow detachment rate for a particle of size class ϕ [m s⁻¹]

 β_{φ} is an empirical parameter reflecting the changing energy arriving at the surface with increasing flow depth (relative to the diameter of a particle of size class φ).

Thirdly, concentrated erosion is assumed to occur with the onset of turbulence, which is taken to occur when the flow Reynolds number, $Re \ge 500$, corresponding to partial turbulence in the transitional flow régime. The results of Parsons *et al.* (1998) suggest that it is important to take the transitional régime rather than full turbulence at $Re \ge 2500$ as the onset of concentrated erosion in hillslope settings. The approach followed is the development of the Einstein (1942) probabilistic approach as modified by Cheng and Chiew (1998) and Wu and Lin (2002), who assume a lognormal distribution of instantaneous velocities. The probability that a particle will be entrained is

$$P_{\varphi} = 0.5 - 0.5 \frac{\ln(0.049/(0.25U_{*,\varphi}))}{\left|\ln(0.049/(0.25U_{*,\varphi}))\right|} \sqrt{1 - \exp\left(-\frac{2}{\pi} \left[\frac{\ln(0.049/(0.25U_{*,\varphi}))}{0.702}\right]^{2}\right)}$$
(10)

where $U_{*,\varphi}$ is the dimensionless shear velocity for a particle of size class φ [—] defined as $U_{*,\varphi} = u_*^2/(\sigma g D_{\varphi})$ u_* is the flow shear velocity [m s⁻¹] calculated as $u_* = \sqrt{ghS}$

 σ is the specific density of the sediment [--] calculated as $\sigma = \frac{(\rho_s - \rho)}{\rho}$

g is acceleration due to gravity $[m^2 s^{-1}]$

 D_{φ} is the median particle diameter of size class φ [m]

 ρ is the density of water [kg m⁻³].

Thus, the erosion by concentrated flow is given as

$$\varepsilon_{c,\varphi} = P_{\varphi} \delta_{\varepsilon_c,\varphi} \tag{11}$$

where $\varepsilon_{c,\varphi}$ is detachment rate by concentrated flow for particles of size class φ [m s⁻¹] $\delta_{\varepsilon,\varphi}$ is the effective detachment rate of sediment of size class φ [m s⁻¹].

The parameter $\delta_{\varepsilon,\varphi}$ is included in Equation (11) to allow for the temporal scaling of concentrated erosion rates. Otherwise, for finer particles in high flow rates, where values of P_{φ} approach one, the continuity equation breaks down.

Deposition is modelled using the transport-distance approach. Given the estimate of the amount of erosion and knowledge of the distribution function of travel distances of particles under specific transport mechanisms and flow conditions, the deposition rate can be calculated directly at each point along the transport pathway. In all cases at present, an exponential distribution function is assumed, both because this function captures the principal characteristic of a declining probability of movement further from the source, and because insufficient data exist in order to evaluate whether other, more complex, distribution functions might better fit actual distributions of transport distances. The simplicity of the exponential distribution is complemented by its ease of parameterization, as it has a single parameter that relates directly to the mean or median observed travel distance. Thus, the parameter can also be interpreted in a physically meaningful way.

Four transport modes are simulated. First, splash is simulated using an exponential transport-distance model, which is the same as that used by Wainwright *et al.* (1995, 1999a). Routing of splash is in the four cardinal directions away from the source point, with relative weightings for upslope and downslope movement. Secondly, unconcentrated overland flow is simulated using an exponential distribution parameterized by the median travel distance calculated as

$$L_{u,\omega} = 0.199 R^{1.446} \omega^{1.607} M_{\omega}^{-0.96}$$
(12)

where $L_{u,\varphi}$ is the median travel distance in unconcentrated flows for a particle of size φ [m]

R is rainfall kinetic energy [J m⁻² s⁻¹]

 ω is overland flow energy or stream power [J m⁻² s⁻¹], calculated as $\omega = ghuS$

 M_{φ} is the mass of a particle of size φ [g].

This equation is derived from a reanalysis of the data presented by Parsons *et al.* (1998). Although these data only relate to relatively coarse particle sizes, Parsons *et al.* (2004) found that they predicted rates of movement of finer

particles adequately. The redistribution and deposition of sediment follows the spatial pattern of the flow-routing algorithm directly. Thirdly, concentrated flows are simulated with an exponential distribution function parameterized using the median transport distances based on the review of available data by Hassan *et al.* (1992):

$$L_{c,\varrho} = 2.85 \times 10^{-3} (\omega - \omega_{*,\varrho})^{1.31} D_{\varrho}^{-0.94}$$
⁽¹³⁾

where $L_{c,\varphi}$ is the median travel distance in concentrated flows for a particle of size φ [m]

 $\omega_{*,\varphi}$ is Bagnold's threshold stream power for the initiation of movement of sediments of size class φ [J m⁻² s⁻¹], given as $\omega_{*,\varphi} = 290D_{50}^{1.5} \log(12D_{\varphi}/D_{50})$, where D_{50} is the median grain size of the bed.

It should be noted that this equation is based on the ordinary regression results of Hassan *et al.* (1992), rather than the functional relationship form, because it is more consistent with the results of Parsons *et al.* (1993, 1998) for hillslopes (Figure 2). Fourthly, suspension is calculated once the suspension threshold of Van Rijn (1984) is exceeded. Suspension by this criterion occurs once the flow shear velocity is greater than a critical value, defined as



Figure 2. Comparison of information on (a) travel distances and (b) virtual velocities from Hassan et al. (1992) for concentrated flows and Parsons et al. (1998) for unconcentrated flows. See the text for further discussion.

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$$u_{**,\varphi} = \frac{4v_{\downarrow,\varphi}}{D_{*,\varphi}}, \qquad 1 < D_{*,\varphi} \le 10$$
(14a)

$$u_{**,\varphi} = 0.4v_{\downarrow,\varphi}, \qquad D_{*,\varphi} > 10 \tag{14b}$$

where $u_{**,\varphi}$ is the critical shear velocity for suspension of a particle of size class φ [m s⁻¹]

 $v_{\downarrow,\varphi}$ is the settling velocity of a particle of size class φ [m s⁻¹]

 $D_{*,\varphi}$ is the dimensionless particle size [--], defined as $D_{*,\varphi} = D_{\varphi} \left(\frac{(\sigma - 1)g}{v^2}\right)^{1/3}$, with v the kinematic viscosity of water [m² s⁻¹].

There is little literature available on transport distances of particles in suspension in fluid flows. However, theoretical studies exist of suspension distances in aeolian transport that are physically based, and thus a first approximation can be made by using such models and altering the physical parameters, notably the fluid viscosity. Anderson (1987) produced a model that can be used in this way, based on the effects of turbulent fluctuations of velocity on particle suspension trajectories for varying sediment sizes. Solving the model numerically for a range of particle sizes and simulated flow conditions produces a first approximation for the travel distance of particles suspended in water flows as

$$L_{\rm top} = 7.28 \times 10^2 e^{(7.33 \times 10^{-3} \overline{\sigma})} e^{(-6.127 D_{\rm g})}$$
(15)

where $L_{s,\phi}$ is the median travel distance in suspension for a particle of size ϕ [m].

Once suspension occurs, travel distances thus increase significantly compared to other flow mechanisms, which is compatible with empirical observations (e.g. Partheniades, 1972; Cahill *et al.*, 1974; Piest *et al.*, 1975; Verhoff *et al.*, 1980, 1982; Bonniwell *et al.*, 1999). Routing of sediment movement by any flow process follows the steepest descent algorithm of the flow-routing algorithm. At present, no down-flow changes in transport distance are calculated, except when a cell with no current flow is encountered, in which case all remaining sediment in the flow is deposited (Figure 3).

Calculation of virtual velocities is carried out for the unconcentrated, concentrated and suspension cases. The redistribution of splash is simulated directly as described by Wainwright *et al.* (1995, 1999a), and thus a virtual velocity is not required in this case. The data set of Parsons *et al.* (1998) is used to parameterize the unconcentrated flow case:



Figure 3. Illustration of the sediment-routing algorithms used in MAHLERAN. Sediment detached in the starting cell (labelled 0) is transported and either redeposited in the starting cell, or deposited according to the proportions given by the different traveldistance functions (the integrals denoted 1, 2, 3, 4, ...). In the case of splash transport, the second across-slope and the upslope components have been omitted for clarity.

$$v_{\rm pu,\omega} = 0.525 R^{2.35} \omega^{0.981} M_{\omega}^{-1} \tag{16}$$

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where $v_{p,u,\varphi}$ is the virtual velocity of a particle of size φ in unconcentrated flows [m s⁻¹].

Hassan et al. (1992) give the virtual velocity of particles in concentrated flows as

$$v_{\rm p.c,\omega} = 1.92 \times 10^{-2} (\omega - \omega_{*,\omega})^{1.01} \tag{17}$$

where $v_{p,c,\varphi}$ is the virtual velocity of a particle of size φ in concentrated flows [m s⁻¹].

Note that for compatibility with the travel distances estimated above and with virtual velocities as derived for hillslopes, the normal regression equation from the data of Hassan *et al.* (1992) are again used (Figure 2). As a first approximation, we also follow the approach of Bennett (1974) in assuming that the virtual velocity of a particle in suspension is equal to the mean flow velocity:

$$v_{\mathbf{p},\mathbf{s},\boldsymbol{\varphi}} = u \tag{18}$$

where $v_{p,s,\phi}$ is the virtual velocity of a particle of size ϕ in suspension [m s⁻¹].

Abrahams and Atkinson (1993) have demonstrated that this relationship breaks down for high sediment concentrations, and further analyses will need to be carried out as to the sensitivity of model output to this approximation.

A summary of the algorithm for erosion and deposition conditions, together with the related thresholds, is presented in Figure 4. Solution of Equation (6c) for each size class for each time step can thus be carried out once the



Figure 4. Summary of the calculation algorithm used in MAHLERAN. The main components in terms of detachment and the use of travel distance and virtual velocity to estimate sediment discharge are highlighted.

appropriate erosion and deposition rates have been calculated. The sediment mass-balance Equation (6) is equivalent to a kinematic wave model of sediment transport, and can thus be solved with an appropriate numerical technique. At present, we employ a simple backward-difference (Euler) scheme, as outlined for water flows by Scoging (1992). The combined modelling framework has been called MAHLERAN (Model for Assessing Hillslope-Landscape Erosion, Runoff And Nutrients – the latter component being considered in the work of Müller *et al.* (2007).

Conclusions

The assessment of existing process-based, soil-erosion models suggests that there are a number of important misconceptions about the processes that have derived either from misreadings of the literature, or from implicit simplifications from that literature, or from the propagation of hypotheses as accepted fact. The combination of these misconceptions has led to the production of a generation of erosion models that are flawed in significant ways, leading to their need for excessive calibration to perform as well as simple, empirically based models. An alternative approach is presented here, that derives from the conceptual approach of Parsons *et al.* (2004) and Wainwright *et al.* (2001), and has a more sound representation of the physical processes in order to overcome the limitations of existing models. In the current version of the model these processes are parameterized as far as possible from existing literature sources, which allows the estimation of spatial and temporal variations in erosion on an event basis, accounting for differences in particle size. However, it needs to be recognized that the existing literature provides a comparatively weak base for this parameterization, and that there is an urgent need for a substantial research effort into erosion dynamics in nonsteady-state conditions. In subsequent papers (Wainwright *et al.*, in press a, in press b) we will test the model against empirical data, undertake a sensitivity analysis and assess its scaling characteristics.

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References

- Abrahams AD, Atkinson JF. 1993. Relation between grain velocity and sediment concentration in overland flow. *Water Resources Research* **29**: 3021–3028.
- Al Durrah MM, Bradford JM. 1982. Parameters for describing soil detachment due to single raindrop detachment. Soil Science Society of America Journal 46: 836–840.
- Anderson RS. 1987. A theoretical model for aeolian impact ripples. Sedimentology 34: 943–956.
- Bennett JP. 1974. Concepts of mathematical modelling of sediment yield. Water Resources Research 10: 485-492.
- Beuselinck L, Govers G, Steegen A, Hairsine PB, Poesen J. 1999. Evaluation of the simple settling theory for predicting sediment deposition by overland flow. *Earth Surface Processes and Landforms* 24: 993–1007.
- Beuselinck L, Hairsine PB, Govers G, Poesen J. 2002. Evaluating a single-class net deposition equation in overland flow conditions. *Water Resources Research* **38(7)**. DOI: 10.1029/2001WR000248
- Bonniwell EC, Matisoff G, Whiting PJ. 1999. Determining the times and distances of particle transit in a mountain stream using fallout radionuclides. *Geomorphology* 27: 75–92.
- Bradford JM, Ferris JE, Remley PA. 1987a. Interrill soil erosion processes: I. Effect of surface sealing on infiltration, runoff and soil splash detachment. *Soil Science Society of America Journal* **51**: 1566–1571.
- Bradford JM, Ferris JE, Remley PA. 1987b. Interrill soil erosion processes: II. Relationship of splash detachment to soil properties. *Soil Science Society of America Journal* **51**: 1571–1575.
- Bradford JM, Remley PA, Ferris JE, Santini JB. 1986. Effect of soil surface sealing on splash from a single waterdrop. Science Society of America Journal 50: 1547–1552.
- Cahill TH, Imperato P, Verhoff FH. 1974. Evaluation of phosphorus dynamics in a watershed. *Journal of the Environmental Engineering Division, Proceedings ASCE* 100: 439–458.
- Cheng NS, Chiew YM. 1998. Pickup probability for sediment entrainment. Journal of Hydraulic Engineering ASCE 124: 232–235.
- Cochrane TA, Flanagan DC. 1996. Detachment in a simulated rill. Transactions of the American Society of Agricultural Engineers 40: 111–119.
- Darboux F, Huang C-H. 2005. Does soil-surface roughness increase or decrease water and particle transfers? *Soil Science Society of America Journal* **69**: 748–756.
- Dietrich WE, Whiting PJ. 1989. Boundary shear stress and sediment transport in river meanders of sand and gravel. In *River Meandering*, American Geophysical Union Water Resources Monograph 12, Ikeda S, Parker G (eds). Washington, DC; 1–50.
- Drewry JJ, Paton RJ, Monaghan RM. 2004. Soil compaction and recovery cycle on a Southland dairy farm: implications for soil monitoring. *Australian Journal of Soil Research* **42**: 851–856.

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- Einstein HA. 1942. Formulas for the transport of bed sediment. *Transactions of the American Society of Civil Engineers* **107**: 561–574. Einstein HA. 1950. *The Bed-Load Function for Sediment Transportation in Open Channel Flows*, Technical Bulletin 1026. US Department
- of Agriculture, Soil Conservation Service: Washington, DC.
- Farres PJ. 1987. The dynamics of rainsplash erosion and the role of soil aggregate stability. Catena 14: 119-130.
- Favis-Mortlock D. 1998. Validation of field-scale soil-erosion models using common datasets. In *Modelling Soil Erosion by Water*, Boardman J, Favis-Mortlock D (eds). Springer, Berlin; 89–127.
- Ferro V. 1998. Evaluating overland flow sediment transport capacity. Hydrological Processes 12: 1895–1910.
- Foster GR. 1982. Modelling the erosion process. In *Hydrologic Modelling of Small Watersheds*, Haan CT, Johnson HP, Brakensiek DL (eds). American Society of Agricultural Engineers: St Joseph, MI; 297–379.
- Foster GR. 1990. Process-based modelling of soil erosion by water on agricultural land. In *Soil Erosion on Agricultural Land*, Boardman J, Foster IDL, Dearing JA (eds). Wiley: Chichester; 429–445.
- Foster GR, Meyer LD. 1972. A closed form soil-erosion equation for upland areas. In *Sedimentation*, Shen HW (ed.). Colorado State University: Fort Collins, CO; 12.1–12.19.
- Goodrich DC, Unkrich CL, Smith RE, Woolhiser DA. 2002. KINEROS2 a distributed kinematic runoff and erosion model. Proceedings of the Second Federal Interagency Hydrologic Modeling Conference, July 28th–August 1st 2002, Las Vegas, NV.
- Govindaraju RS, Kavvas ML. 1991. Modelling the erosion process over steep slopes: approximate analytical solutions. *Journal of Hydrology* **127**: 279–305.
- Guy HP, Simons DB, Richardson EV. 1966. Summary of Alluvial Channel Data from Flume Environments, 1956–1961, USGS Professional Paper 462-I. United States Geological Survey: Washington, DC.
- Hassan MA, Church M, Ashworth PJ. 1992. Virtual rate and mean distance of travel of individual clasts in gravel-bed channels. *Earth Surface Processes and Landforms* 17: 617–627.
- Huang C, Wells LK, Norton LD. 1999. Sediment transport capacity and erosion processes: model concepts and reality. *Earth Surface Processes and Landforms* 24: 503–516.
- Hubbell DW, Sayre WW. 1964. Sand transport studies with radioactive tracers. Journal of the Hydraulics Division, ASCE 90: 39-68.
- Jetten V. 2003. LISEM: Limburg Soil Erosion Model, Windows version 2.x user manual. http://www.itc.nl/lisem [Accessed 10 December 2007].

Jetten V, de Roo A, Favis-Mortlock D. 1999. Evaluation of field-scale and catchment-scale soil erosion models. Catena 37: 521-541.

Kirkby MJ. 1978. Implications for sediment transport. In Hillslope Hydrology, Kirkby MJ (ed.). Wiley: Chichester; 325–363.

- Kirkby MJ. 1991. Sediment travel distance as an experimental and model variable in particulate movement. In *Erosion, Transport and Deposition Processes*, Bork H-R, de Ploey J, Schick AP (eds). *Catena Supplement* **19**: 111–128.
- Kirkby MJ. 1992. An erosion-limited hillslope evolution model. In Functional Geomorphology: Landform Analysis and Models, Schmidt K-H, de Ploey J (eds). Catena Supplement 23: 157–187.

Laflen JM, Lane LJ, Foster GR. 1991. A new generation of erosion-prediction technology. Journal of Soil and Water Conservation 46: 34-38.

- Lane LJ, Shirley ED, Singh VP. 1988. Modelling erosion on hillslopes. In *Modelling Geomorphological Systems*, Anderson MG (ed.). Wiley: Chichester; 287–308.
- Lisle IG, Rose CW, Hogarth WL, Hairsine PB, Sander GC, Parlange JY. 1998. Stochastic sediment transport in soil erosion. *Journal of Hydrology* **204**: 217–230.
- Merten GH, Nearing MA, Borges ALO. 2001. Effect of sediment load on soil detachment and deposition in rills. Soil Science Society of America Journal 65: 861–868.
- Meyer LD, Monke EJ. 1965. Mechanics of soil erosion by rainfall and overland flow. *Transactions of the American Society of Agricultural Engineers* 8: 572–577.
- Mitas L, Mitasova H. 1998. Distributed soil erosion simulation for effective erosion prevention. Water Resources Research 34: 505-516.
- Morgan RPC, Quinton JN, Smith RE, Govers G, Poesen JWA, Auerswald K, Chisci G, Torri D, Styczen ME. 1998. The European Soil Erosion Model (EUROSEM): a dynamic approach for predicting sediment transport from fields and small catchments. *Earth Surface Processes and Landforms* 23: 527–544.
- Müller EN, Wainwright J, Parsons AJ. 2007. The stability of vegetation boundaries and the propagation of desertification in the American Southwest: a modelling approach. *Ecological Modelling* **208**: 91–101.
- Nearing MA, Bradford JM. 1985. Single waterdrop splash detachment and mechanical properties of soils. *Science Society of America Journal* **49**: 547–552.
- Parker G. 2007. Transport of gravel and sediment mixtures. In *Sedimentation Engineering: Theories, Measurements, Modeling and Practice*, Garcia M (ed.). ASCE Manuals and Reports on Engineering Practice No. 110. Reston, VA.
- Parsons AJ, Abrahams AD. 1992. Preface. In *Overland Flow: Hydraulics and Erosion Mechanics*, Parsons AJ, Abrahams AD (eds). University College London Press: London; i–xiii.
- Parsons AJ, Brazier RE, Wainwright J, Powell DM. 2006a. Scale relationships in hillslope runoff and erosion. *Earth Surface Processes and Landforms* **31**: 1384–1393.
- Parsons AJ, Stromberg SGL, Greener M. 1998. Sediment-transport competence of rain-impacted interrill overland flow. *Earth Surface Processes and Landforms* 23: 365–375.
- Parsons AJ, Wainwright J, Abrahams AD. 1993. Tracing sediment movement on semi-arid grassland using magnetic susceptibility. *Earth Surface Processes and Landforms* 18: 721–732.
- Parsons AJ, Wainwright J, Abrahams AD, Simanton JR. 1997. Distributed dynamic modelling of interrill overland flow. *Hydrological Processes* **11**: 1833–1859.
- Parsons AJ, Wainwright J, Brazier RE, Powell DM. 2006b. Is sediment delivery a fallacy? *Earth Surface Processes and Landforms* **31**: 1325–1328.

- Parsons AJ, Wainwright J, Powell DM, Kaduk J, Brazier RE. 2004. A conceptual model for understanding and predicting erosion by water. *Earth Surface Processes and Landforms* 29: 1293–1302.
- Partheniades E. 1972. Results of recent investigations on erosion and deposition of cohesive soils. In *Sedimentation*, Shen HW (ed.). Colorado State University: Fort Collins, CO; 21.1–21.39.
- Piest RF, Kramer LA, Heinemann HG. 1975. Sediment movement from loessial watersheds. In *Present and Prospective Technology for Predicting Sediment Yields and Sources*, US Department of Agriculture Publication ARS-S-40; 130–141.

Powell DM. 1998. Patterns and processes of sediment sorting in gravel-bed rivers. Progress in Physical Geography 22: 1-32.

- Quansah C. 1981. The effect of soil type, slope, rain intensity and their interactions on splash detachment and transport. *Journal of Soil Science* **32**: 215–224.
- Rejman J, Usowicz B, Debicki R. 1999. Source of errors in predicting silt soil erodibility with USLE. Polish Journal of Soil Science 32: 13–22.
- Rice CT, Wilson BN. 1990. Analysis of Dynamic Variables of Rill Flow, ASAE Paper 902011 (Presentation at the 1990 International Summer Meeting, Columbus, OH, 1990). ASAE: St. Joseph, MI.
- Risse LM, Nearing MA, Lui BY, Zhang XC, Baffaut C, Laflen JM. 1995. WEPP: validation and applications. Carrying the torch for erosion control an Olympic task. *Proceedings of the XXVI Conference*, Atlanta. IECA; 471–486.
- Rose CW. 1985. Developments in soil erosion and deposition models. Advances in Soil Science 2: 2-63.
- Rose CW, Williams JR, Sander GC, Barry DA. 1983. A mathematical model of soil erosion and deposition processes: I. Theory for a plane land element. *Soil Science Society of America Journal* **47**: 991–995.
- Scoging HM. 1992. Modelling overland-flow hydrology for dynamic hydraulics. In *Overland Flow: Hydraulics and Erosion Mechanics*, Parsons AJ, Abrahams AD (eds). University College London Press: London; 89–104.
- Smith RE, Goodrich DC, Woolhiser DA, Unkrich CL. 1995. KINEROS a kinematic runoff and erosion model. In Computer Models of Watershed Hydrology, Singh VP (ed.). Water Resources: Littleton, CO; 697–732.
- Smith RE, Parlange J-Y. 1978. A parameter-efficient hydrologic infiltration model. Water Resources Research 14: 533-538.

Tisdale JM, Oades JM. 1982. Organic matter and water-stable aggregates in soils. Journal of Soil Science 33: 141-163.

- Tiwari AK, Risse LM, Nearing MA. 2000. Evaluation of WEPP and its comparison with USLE and RUSLE. *Transactions of the ASAE* **43**: 1129–1135. Torri D. 1987. A theoretical study of soil detachability. *Catena Supplement* **10**: 15–20.
- Torri D, Biancalani R, Poesen J. 1990. Initiation of motion of gravels in concentrated overland flow: cohesive forces and probability of entrainment. *Catena Supplement* 17: 79–90.
- Torri D, Sfalanga M, del Sette M. 1987. Splash detachment: runoff depth and soil cohesion. Catena 14: 149-155.
- Van Rijn LC. 1984. Sediment transport part II: suspended load transport. Journal of Hydraulic Engineering 110: 1613–1641.
- Verhoff FH, Melfi DA, Yaksich SM. 1982. An analysis of total phosphorus transport in river systems. Hydrobiologia 91: 241-252.
- Verhoff FH, Yaksich SM, Melfi DA. 1980. River nutrient and chemical transport estimation. *Journal of the Environmental Engineering Division ASCE* 106: 591–608.
- Wainwright J, Parsons AJ. 2002. The effect of temporal variations in rainfall on scale dependency in runoff coefficients. *Water Resources Research* **38**(12): 1271. DOI: 10.1029/2000WR000188
- Wainwright J, Parsons AJ, Abrahams AD. 1995. A simulation study of the role of raindrop erosion in the formation of desert pavements. *Earth Surface Processes and Landforms* **20**: 277–291.
- Wainwright J, Parsons AJ, Abrahams AD. 1999a. Field and computer simulation experiments on the formation of desert pavement. *Earth Surface Processes and Landforms* 24: 1025–1037.
- Wainwright J, Parsons AJ, Powell DM, Brazier RE. 2001. A new conceptual framework for understanding and predicting erosion by water from hillslopes and catchments. In *Soil Erosion Research for the 21st Century. Proceedings of the International Symposium*, Ascough JC II, Flanagan DC (eds). American Society of Agricultural Engineers: St Joseph, MI; 607–610.
- Wainwright J, Parsons AJ, Müller N, Brazier RE, Powell DM, Fenti B. In press a. A transport-distance approach to scaling erosion rates: 2. Sensitivity and evaluation of MAHLERAN. *Earth Surface Processes and Landforms*. DOI: 10.1002/esp1622
- Wainwright J, Parsons AJ, Müller N, Brazier RE, Powell DM, Fenti B. In press b. A transport-distance approach to scaling erosion rates: 3. Evaluating scaling characteristics of MAHLERAN. *Earth Surface Processes and Landforms*. DOI: 10.1002/esp1623
- Wainwright J, Thornes JB. 1991. Computer and hardware simulations of archæological sediment transport. In *Computer Applications and Quantitative Methods in Archæology 1990*, BAR International Series 565, Lockyear K, Rahtz S (eds). Oxford; 183–194.
- Willis JC. 1971. Erosion by Concentrated Flow, United States Department of Agriculture, Agricultural Research Service Paper ARS 41-179. USDA-ARS: Beltsville, MA.
- Woolhiser DA, Smith RE, Goodrich DC. 1990. KINEROS, a Kinematic Runoff and Erosion Model: Documentation and User Manual, US Department of Agriculture, Agricultural Research Service Publication ARS-77. Washington, DC.
- Wu FC, Lin YC. 2002. Pickup probability of sediment under log-normal velocity distribution. Journal of Hydraulic Engineering ASCE 128: 438–442.
- Yalin MS. 1972. Mechanics of Sediment Transport. Pergamon: Oxford.
- Yang CT, Sayre WW. 1971. Stochastic model for sand dispersion. *Proceedings of the American Society of Civil Engineers, Journal of the Hydraulics Division* **97**(HY2): 265–288.

Yu B. 2003. A unified framework for water erosion and deposition equations. Soil Science Society of America Journal 67: 251–257.

- Zhang XC, Nearing MA, Risse LM, McGregor KC. 1996. Evaluation of WEPP runoff and soil loss predictions using natural runoff plot data. *Transactions, ASAE* 39: 855–863.
- Zheng F-L, Huang C-H, Norton LD. 2000. Vertical hydraulic gradient and run-on water and sediment effects on erosion processes and sediment regimes. *Soil Science Society of America Journal* **64**: 4–11.
- Zingg AW. 1940. Degree and length of land slope as it affects soil loss in runoff. Agricultural Engineering 21: 59-64.