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Partitioning resistance to overland flow on rough mobile beds

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Abstract

For overland flows transporting predominantly bed load over rough mobile beds without rainfall, resistance to flow f may be divided into four components: surface resistance f_s, form resistance f_f, wave resistance f_w, and bed-mobility resistance f_m. In this study it is assumed that $f = f_s + f_f + f_w + f_m$, and an equation is developed for each component. The equations for f_s and f_f are borrowed from the literature, while those for f_w and f_m are developed from two series of flume experiments in which the beds are covered with various concentrations of large-scale roughness elements. The first series consists of 65 experiments on fixed beds, while the second series contains 194 experiments on mobile beds. All experiments were performed on the same slope ($S = 0{\cdot}114$) and with the same size of sediment ($D = 0{\cdot}00074$ m). The equations for f_w and f_m are derived by a combination of dimensional analysis and regression analysis. The analyses reveal that the major controls of f_w and f_m are the Froude number F and the concentration of the roughness elements C_r. When the equations for f_w and f_m are summed, the C_r terms cancel out, leaving $f_{w+m} = 0{\cdot}63F^{-2}$. An equation is developed that predicts total f, and the contributions of f_s, f_f, f_w and f_m to f are computed from the series 1 and 2 experiments. An analysis of the first series reveals that in clear-water flows over fixed beds, f_w accounts for 52 per cent of f. A similar analysis of the second series indicates that in sediment-laden flows over mobile beds f_w comprises 37 per cent and f_m 32 per cent of f, so that together f_w and f_m account for almost 70 per cent of f. Finally, regression analyses indicate that where $F > 0{\cdot}5$, f_w and f_m each vary with F^{-2} and $f_w/f_m = 1{\cdot}18$. The equation developed here for predicting total f applies only to the range of hydraulic, sediment, and bed roughness conditions represented by the experimental data. With additional data from a broader range of conditions the same methodology as employed here could be used to develop a more general equation. Copyright © 2006 John Wiley & Sons, Ltd.

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Introduction

On natural dryland hillslopes, interrill overland flow occurs as a shallow sheet of water containing threads of deeper, faster flow diverging and converging around obstacles such as microtopographic highs, rocks, plants and litter. Such obstacles are referred to as large-scale roughness elements because they protrude through the flow and impede its passage downslope. By retarding the flow, the roughness elements contribute to flow resistance herein measured by the Darcy–Weisbach friction factor:

$$f = 8gSh/u^2 \tag{1}$$

where g is the acceleration of gravity (m s^{-2}), S is the energy slope, h is the mean flow depth (m), and u is the mean flow velocity (m s^{-1}). The Darcy–Weisbach friction factor is dependent on flow rate and is therefore often related to the flow Reynolds number:

$$Re = 4uh/\nu \tag{2}$$

where ν is the kinematic viscosity of the fluid ($m^2\ s^{-1}$).

Where overland flow is undisturbed by rainfall and has a mobile bed and large-scale roughness elements, the total resistance f may be divided into four components which are assumed to be additive but not necessarily independent. Thus:

$$f = f_s + f_f + f_w + f_m \tag{3}$$

where f_s = surface resistance, f_f = form resistance, f_w = wave resistance and f_m = bed-mobility resistance.

Surface resistance is imparted by the bed and the submerged sides of the large-scale roughness elements. Traditionally, surface resistance has been viewed as the kinetic energy dissipated in overcoming the no-slip condition at the boundary. But where the flow is turbulent, it also includes energy dissipated in separation eddies generated by surface grains that protrude through the viscous sublayer at the base of the flow. Form resistance is created by obstacles that protrude into or through the flow, thereby giving rise to spatially varied cross-sections and/or changing flow directions, which cause energy to be dissipated in separation eddies, secondary circulation, and locally increased shear (Abrahams *et al*., 1994). Wave resistance refers to the energy dissipated in maintaining an uneven water surface around protruding roughness elements (Abrahams and Parsons, 1994). Bed-mobility resistance refers to all processes associated with a moving bed that contribute to flow resistance. These processes include bed-load transport, bed deformation and bed elasticity. In an earlier paper (Hu and Abrahams, 2004) we were able to isolate bed-load transport resistance from the other processes associated with bed-mobility resistance and analyse it separately. Thus, in the present paper, bed-mobility resistance includes bed-load transport resistance, whereas in the previous one it did not.

Since Emmett's (1970) pioneering work, considerable progress has been made in the analysis and modelling of the effect of large-scale roughness on resistance to overland flow. Hirsch (1996) developed a flow resistance model (outlined by Abrahams *et al*., 1992) for predicting f based on separate equations for f_s, f_f and f_w that apply where clear water flows over rough fixed beds. This model is defined by:

$$f = [(3{\cdot}19\ Re^{-0{\cdot}45}) + ((4C_d \Sigma A_i)/A_b)]e^{6{\cdot}45C_r} \tag{4}$$

total resistance	grain resistance	form resistance	wave resistance

where A_i is the wetted cross-sectional area of the ith element (m^2), A_b is the area of the flume bed (m^2), C_d is the drag coefficient, and C_r is the concentration of the roughness elements. This model is neither additive nor multiplicative but a hybrid of the two: $f = (f_s + f_f)f_w$. Application of Hirsch's model reveals that where the Froude number $F = u/(gh)^{0{\cdot}5} > 0{\cdot}5$, wave resistance increases with C_r and dominates the resistance to flow wherever $C_r > 0{\cdot}1$.

Lawrence (1997) formulated a model for predicting f for clear-water flows over fixed beds with various concentrations of hemispheres. Implicit in the derivation of this model is the assumption that f consists entirely of form resistance, i.e.

$$f = f_f = 8/\pi C_r\, C_d \min[\pi/4,\ h/k] \tag{5}$$

where min $[x, y]$ denotes the smaller of the values x and y. In Equation 5, $f \propto h/k$, where k is the height of the roughness elements. Because Lawrence (1997) used hemispheres as roughness elements, $k = b/2$, where b is the diameter of the sphere. Thus f is actually a function of $h/(b/2)$ rather than h/k. Where the roughness elements are cylinders that protrude through the flow (as in the present study), Equation 5 becomes (Abrahams, 1998):

$$f = f_f = \frac{16}{\pi} C_r C_d \frac{h}{b} \tag{6}$$

However, when $h/b \le 1$ and $C_d = 1{\cdot}2$ (the conventional value of C_d for an isolated cylinder), the right-hand side of Equation 5 underestimates f calculated using Equation 1. Lawrence (2000) showed that this underestimation can be corrected by increasing C_d and that C_d is negatively correlated with h/b, which is positively correlated with the deformation of the water surface and, hence, wave resistance. Thus Lawrence (2000), like Hirsch (1996) and Abrahams *et al*. (1992), concluded that wave resistance is the dominant component of flow resistance in overland flow on rough fixed beds.

The flume studies summarized above all investigated the effect of bed roughness on flow resistance by placing various concentrations of large-scale roughness elements on a fixed bed. A failing of these studies is that they neglect the effect of bed mobility on flow resistance. Accordingly, the purpose of this study is to evaluate the effect of bed mobility on resistance to overland flow on rough beds (i.e. beds covered with large-scale roughness elements). This is done by analysing two series of flume experiments with similar hydraulic and bed roughness characteristics. In the first series the water is free of sediment and flows over a fixed bed, whereas in the second the water is laden with sediment and flows over a mobile bed. The data collected during these experiments are recorded in two Appendices which are located at the following Internet address: http://www.esu.edu/~shu/appendices1&2.htm. These data are summarized in Table I.

Methods

The first series of experiments was performed by Hirsch (1996) in a flume 0·5 m wide and 4·9 m long with Plexiglas walls and a smooth aluminium bed covered with cylinders which served as large-scale roughness elements. These cylinders have diameters D_r of 0·020 and 0·031 m and concentrations C_r ranging from 0·02 to 0·24. In these experiments the mean flow depth h was determined by using a Vernier point gauge to measure the distance from the bed to the water surface. Measurements were made at 50 randomly selected locations and then averaged to arrive at the mean flow depth. Water discharge Q_w ($m^3 s^{-1}$) was determined by measuring the time it took for the outflow to fill a bucket of known volume. The mean flow velocity u was then obtained from $u = Q_w/Wh$, where W is the flume width (m).

Hirsch performed a total of 83 experiments, 65 of which are included in this study because they have the following properties: (1) $k > h$, (2) $F > 0{\cdot}5$ and (3) $Re > 2000$. These properties ensure: (1) that the roughness elements protrude through the flow; (2) that f, which is dominated by wave resistance, is inversely related to F (e.g. Hsieh, 1964; Flammer *et al*., 1970; Abrahams and Parsons, 1994); and (3) that the flow over the rough bed is turbulent.

The second series of 194 experiments is a subset of a much larger data set collected by Abrahams *et al*. (2001). These experiments were selected because their hydraulics and bed roughness characteristics are similar to those in the series 1 experiments. The flume used in the series 2 experiments is 0·4 m wide and 5·2 m long with Plexiglas walls and an aluminium floor covered with a layer of loose sand approximately 0·015 m thick. This sand has a median diameter D of 0·00074 m and is well sorted with a standard deviation of $(D_{84}/D_{16})^{0.5} = 1{\cdot}08$. Like series 1, the series 2 experiments employed cylinders as large-scale roughness elements. Two sizes of cylinder ($D_r = 0{\cdot}0216$ and 0·0317 m) were used in the experiments, while concentrations C_r ranged from 0·04 to 0·26. These sizes and concentrations of roughness elements are almost identical to those employed in the series 1 experiments.

Water entered the second flume by overflowing from a head tank. The inflow rate Q_w ($m^3 s^{-1}$) was measured by a flow meter located on the inlet pipe to the tank. The volumetric sediment discharge Q_s ($m^3 s^{-1}$) was determined by sampling the water–sediment mixture leaving the flume, weighing the water and sediment in each sample, and converting the weights to volumes by multiplying by the water and sediment densities of $\rho = 1000$ kg m^{-3} and $\rho_s = 2650$ kg m^{-3}, respectively. The volumetric sediment concentration C was then obtained from:

$$C = Q_s/(Q_w + Q_s) \tag{7}$$

Mean flow velocity u was determined by a salt tracing technique described by Li and Abrahams (1997, 1999). Knowing $Q = Q_w + Q_s$ and u, mean flow depth h was calculated from $h = Q/uw$, where w is the effective flow width:

$$w = W(1 - C_r) \tag{8}$$

Table I. Variable ranges in two series of experiments

Series	h ($\times10^{-2}$ m)	u (m s^{-1})	v (10^{-4} $m^2 s^{-1}$)	F	Re	D_r (10^{-2} m)	C_r (m)	C	D (10^{-3} m)	S
1*	0·20–1·86	0·16–0·79	0·0095–0·0166	0·51–2·81	2028–28 380	3·1, 2·0	0·02–0·24	–	–	0·114
2	0·16–0·43	0·28–1·99	0·0100–0·0123	0·50–1·61	2045–17 390	3·165, 2·155	0·04–0·26	0·016–0·04	0·74	0·114

* The data in this series are available in Hirsch (1996).

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The kinematic viscosity of the water ν ($m^2 s^{-1}$) was obtained from its temperature.

In overland flow sediment may be transported as bed load or suspended load. Because these two modes of transport affect flow resistance differently (Bridge, 2003, pp. 55–67), we confined the series 2 experiments to flows in which $P_b \geq 0{\cdot}85$, where P_b is the proportion of the total sediment load that moves as bed load. In other words, we restricted the study to flows that were transporting predominantly bed load. The value of P_b was calculated by estimating the bed-load transport rate q_b ($m^2 s^{-1}$) for each flow and then expressing this rate as a proportion of the measured total sediment transport rate. The bed-load transport rate in overland flow on rough surfaces can be estimated using Abrahams and Gao's equation (Abrahams *et al*., 2001):

$$\phi_b = \theta^{1{\cdot}5}\left(1 - \frac{\theta_c}{\theta}\right)^{3{\cdot}4}\frac{u}{u_*} \tag{9}$$

where $\phi_b = q_b/[(g\Delta D)^{0{\cdot}5}D]$ (Einstein, 1950), $\Delta = (\rho_s - \rho)/\rho$ is dimensionless sediment density, ρ_s is the sediment density (kg m^{-3}), $u_* = (ghS)^{0{\cdot}5}$ is the shear velocity (m s^{-1}), $\theta = hS/\Delta D$ is the dimensionless bed shear stress, and the subscript c denotes the critical value at which bed load begins to move. The bed-load transport rate q_b obtained from Equation 9 is then compared with the total-load transport rate q_t ($m^2 s^{-1}$) calculated using:

$$q_t = CQ_w/w \tag{10}$$

In the series 2 experiments in which the bed is rough and mobile, q_b/q_t ranges from 0·85 to 1·40 with a mean of 1·09. Thus, it seems fair to conclude that during the experiments in the second series most, if not all, of the sediment is travelling as bed load.

In both series of experiments the bed is inclined at a slope β of 5·5°, and the effect of the downslope component of gravity on sediment transport is taken into account by replacing $\sin\beta$ with $S = \sin\beta \tan\alpha/[\cos\beta(\tan\alpha - \tan\beta)]$, in which α is the angle of repose, which is generally assumed to be about 32° (van Rijn, 1993; Abrahams *et al*., 2001). In the present study where $\beta = 5{\cdot}5°$, $\sin\beta = 0{\cdot}096$, and corrected $\sin\beta = 0{\cdot}096 \times 1{\cdot}188 = 0{\cdot}114$.

Table I shows that the series 1 and series 2 experiments have identical slopes and similar ranges of hydraulic and bed roughness characteristics. The principal difference between these two series is that the series 2 experiments have mobile beds while the series 1 experiments do not. Inasmuch as $f = f_s + f_f + f_w$ in the series 1 experiments and $f = f_s + f_f + f_w + f_m$ in the series 2 experiments, f_m may be estimated by subtracting the former equation from the latter. However, before f_m can be estimated in this manner, it is necessary to obtain equations for predicting f_s, f_f and f_w. In what follows, we select equations for predicting f_s and f_f from the literature, and we develop an equation for predicting f_w from the series 1 experiments.

Surface Resistance

In this study the surface resistance f_s is calculated by two different methods. The first method is used where the bed is smooth (as in the series 1 experiments), while the second is employed where the bed is granular (as in the series 2 experiments).

For smooth beds f_s is computed using:

$$f_s = 3{\cdot}19\ Re^{-0{\cdot}45} \tag{11}$$

This equation was obtained by Hirsch (1996) by regressing f_s against Re for 20 flows on smooth, fixed plane beds in the same flume, at the same slope, and with a similar range of Re values as the series 1 experiments. It is emphasized that this equation is not a valid predictor of f_s beyond the range of hydraulic and boundary conditions covered by the series 1 experiments (Table I). The values of f_s for these experiments are listed in Appendix 1. The ratio f_s/f ranges from 0·018 to 0·723 and averages 0·186 (Table II).

For granular beds, f_s is obtained from the Savat (1980) algorithm. Savat developed an algorithm that computes f_s using slope, temperature, unit discharge, and the 90th percentile of the grain size distribution D_{90}. This algorithm was originally created to compute the hydraulic properties of sediment-free overland flows on plane fixed beds. G. Govers refined this algorithm and kindly provided us with a copy of the code in Pascal. Extensive testing of the algorithm for flows up to 0·02 m deep (e.g. Govers and Rauws, 1986; Rauws, 1988; Everaert, 1991; Takken and Govers, 2000; Hu and Abrahams, 2004, 2005) has confirmed its accuracy.

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Table II. Proportion of total flow resistance accounted for by each resistance component

	Proportion of total flow resistance f	
Resistance components	**Fixed bed**	**Mobile bed**
f_s	0·186	0·128
f_f	0·294	0·178
f_w	0·520	0·372
f_m	–	0·322
f_{w+m}	–	0·694

In the series 2 experiments f_s is calculated by:

$$f_s = \frac{8\tau_s}{\rho u_s^2} \tag{12}$$

where τ_s is the surface shear stress (N m^{-2}) and u_s (m s^{-1}) is the calculated hydraulic velocity provided by the Savat algorithm. The values of f_s in the series 2 experiments are listed in Appendix 2. The ratio f_s/f ranges from 0·022 to 0·422 and averages 0·128 (Table II).

Form Resistance

The total form drag exerted by all the roughness elements in an area A_B (m^2) is given by:

$$\Sigma F_D = C_d \rho u^2 \Sigma A_F/2 \tag{13}$$

where ΣA_F is the sum of the inundated frontal areas of the roughness elements (m^2). Hirsch (1996) showed that Equations 1 and 13 can be combined to provide a simpler equation for form resistance:

$$f_f = 4C_d \, \Sigma A_F/A_B \tag{14}$$

where $A_F = D_r h$. Abrahams (1998) subsequently rewrote Equation 14 in the form:

$$f_f = (16/\pi)C_d \, (h/D_r)C_r \tag{15}$$

for cylindrical roughness elements, where C_d is assumed to be 1·2 (Petryk, 1969, p. 43; Li and Shen, 1973).

Equation 15 is used to calculate f_f for all experiments in this study (Appendices 1 and 2). For the series 1 experiments (in which clear water flows over a cylinder-covered fixed bed), the ratio f_f/f ranges from 0·066 to 0·609 and averages 0·294 (Table II). For the series 2 experiments (in which sediment-laden water flows over a cylinder-covered mobile bed), the ratio f_f/f ranges from 0·030 to 0·332 and averages 0·179 (Table II). Clearly, the values of f_f/f for the flows over rough fixed beds are greater than those over rough mobile beds, implying that bed mobility reduces form resistance. This finding comes as no surprise as horseshoe vortices scour on the upstream side and deposit on the downstream side of most if not all roughness elements (e.g. Bunte and Poesen, 1994). The resulting deformation of the bed streamlines the flow, causing form drag and hence f_f to decline. At the same time, bed deformation reduces irregularities in the water surface, causing f_w to shrink. In the following section we calculate f_w and then examine its contribution to flow resistance on rough mobile beds.

Wave Resistance

Calculation of f_w

In contrast to Hirsch's (1996) model (i.e. Equation 4), where $(f_s + f_f)$ is multiplied by $e^{6{\cdot}45Cr}$ to obtain f, in the following analysis all the components of f are assumed to be additive. Thus, for sediment-free overland flows on rough fixed

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beds, $f = f_s + f_f + f_w$. Wave resistance f_w is then calculated for each experiment in the first series by subtracting f_s and f_f from f. The calculated values of f_w are listed in Appendix 1. The ratio f_w/f ranges from 0·176 to 0·743 and averages 0·520 (Table II).

Predictive equation for f_w

The calculated values of f_w in the first series of experiments may now be used to develop a predictive equation for f_w. The relevant predictive variables in this equation are identified by dimensional analysis, and the coefficients are determined by multiple regression analysis.

All the basic variables in the functional relation for f_w are as follows:

$$f_w = \Phi(\rho, \mu, g, h, u_*, u, S, C_r, D_r) \tag{16}$$

where μ is dynamic viscosity of the fluid (N s m^{-2}). Selecting ρ, u_* and D_r as the repeating variables and applying the Π-theorem leads to:

$$f_w = \Phi\left(\frac{h}{D_r}, F, R_r, C_r, S\right) \tag{17}$$

where the roughness (or cylinder) Reynolds number $R_r = uD_r/\nu$.

Given that S is a constant in this study, Equation 17 may be written in the form of a power function:

$$f_w = a\left(\frac{h}{D_r}\right)^b F^c R_r^d C_r^e \tag{18}$$

The coefficients a, b, c, d and e are then evaluated by performing a multiple regression analysis on the data from the first series of experiments. The derived regression equation is:

$$f_w = 85{\cdot}31\left(\frac{h}{D_r}\right)^{0{\cdot}250} F^{-0{\cdot}505} R_r^{-0{\cdot}328} C_r^{1{\cdot}035} \tag{19}$$

with $R^2 = 0{\cdot}927$ and the standard error of estimate $SEE = 0{\cdot}136$ log units. The standardized (beta) coefficients for h/D_r, F, R_r and C_r are 0·126, −0·221, −0·147 and 0·671, respectively. The largest beta coefficients are associated with C_r and F, indicating that these variables are the main controls of f_w.

The exponents on h/D_r, F, R_r and C_r in Equation 19 are not significantly different from 0·25, −0·5, −0·33 and 1, respectively. Consequently, the exponents in Equation 19 are changed to these values, and the intercept is recomputed using non-linear regression. The functional relation then becomes:

$$f_w = 79{\cdot}38\left(\frac{h}{D_r}\right)^{0{\cdot}25} F^{-0{\cdot}5} R_r^{-0{\cdot}33} C_r \tag{20}$$

with $R^2 = 0{\cdot}931$ and $SEE = 0{\cdot}127$ log units. Figure 1 shows that Equation 20 is an unbiased estimator of f_w. Equation 20 may now be combined with Equations 11 and 15 to predict f for sediment-free overland flow on rough fixed beds.

Given that C_r and F are the dominant controls of f_w, we explored the possibility of developing a simpler equation for predicting f_w that utilizes just C_r and F as the predictive variables. This was done by retaining the exponents 1 and −0·5 on C_r and F, and then employing non-linear regression analysis to determine the value of the intercept. The derived equation is:

$$f_w = 3{\cdot}32 F^{-0{\cdot}5} C_r \tag{21}$$

with $R^2 = 0{\cdot}88$ and $SEE = 0{\cdot}166$ log units (Figure 2). R^2 is smaller and SEE is larger for Equation 21 than for Equation 20. However, the differences are modest, and a comparison of Figures 1 and 2 suggests that both equations give

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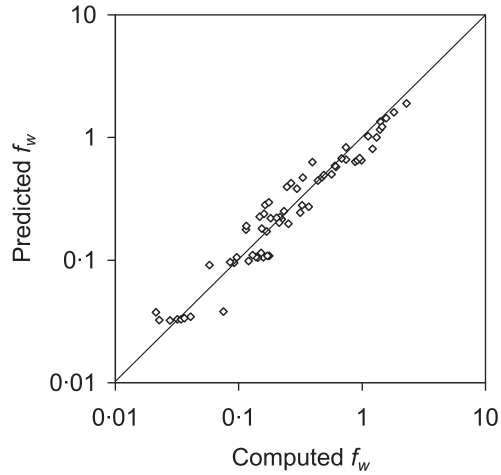

Figure 1. Comparison of computed f_w with f_w predicted by Equation 20.

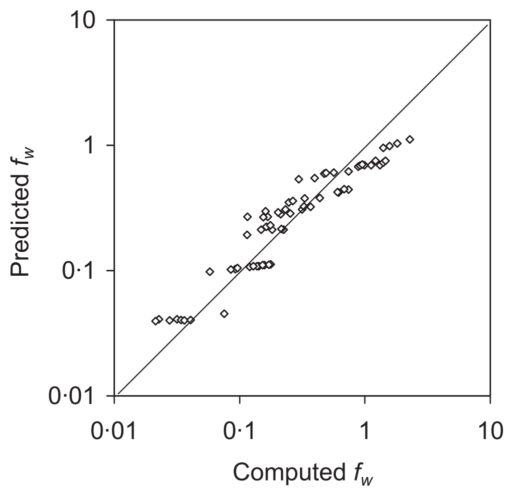

Figure 2. Comparison of computed f_w with f_w predicted by Equation 21.

unbiased estimates of f_w. The salient point here is that Equation 20 can be replaced by Equation 21 without introducing any bias into the model.

Prediction of f_w on rough mobile beds

The second series of experiments represents resistance to flow over rough mobile beds. The values of f_w for mobile beds may be calculated using Equation 21. The calculated values of f_w for the second series are listed in Appendix 2. The ratio f_w/f ranges from 0·117 to 0·564 and averages 0·363 (Table II). Comparison of the mean values of f_w/f for the first and second series indicates that f_w accounts for a larger proportion of f where the bed is fixed (i.e. f_w/f averages 0·520) than where it is mobile (i.e. f_w/f averages 0·363) (Table II). This is because where the bed is mobile, the flow around the roughness elements forms horseshoe vortices that deform the bed, streamline the flow, flatten the water surface, and reduce f_w.

Bed-Mobility Resistance

Calculation of f_m

Given the assumption that the components of f are additive, for rough mobile beds $f = f_s + f_f + f_w + f_m$. The value of f_m is then calculated for each experiment in the second series by subtracting f_s, f_f and f_w from f, where f_s, f_f and f_w are obtained from Equations 12, 15 and 21, respectively. The calculated values of f_m are listed in Appendix 2. The ratio f_m/f ranges from 0·088 to 0·711 and averages 0·322 (Table II).

Predictive equation for f_m

The computed values of f_m in the second series of experiments may now be used to develop a predictive equation for f_m. The relevant predictive variables in this equation are identified by dimensional analysis, and the coefficients are determined by multiple regression analysis.

The initial functional relation for f_m containing all the basic variables for the dimensional analysis is given by:

$$f_w = \Phi(\rho, \mu, g, h, u_*, u, S, C_r, D_r, C_s, D) \tag{22}$$

Selecting ρ, u_* and D_r as the repeating variables and applying the Π-theorem yields:

$$f_m = \Phi\left(\frac{h}{D_r}, F, R_r, C_r, C_s, S, \frac{D}{D_r}\right) \tag{23}$$

Given that S is a constant in this study, Equation 23 may be written in the form of a power function:

$$f_m = a\left(\frac{h}{D_r}\right)^b F^c R_r^d C_r^e C_s^f \left(\frac{D}{D_r}\right)^g \tag{24}$$

The coefficients a, b, c, d, e, f and g are then determined by performing a multiple regression analysis on the data from the second series of experiments. The derived regression equation is:

$$f_m = 0{\cdot}287 F^{-3{\cdot}688} R_r^{-0{\cdot}086} C_r^{-0{\cdot}804} C^{0{\cdot}340} \tag{25}$$

with $R^2 = 0{\cdot}955$ and $SEE = 0{\cdot}073$ log units. The standardized (beta) coefficients for F, R_r, C_r and C are −1·478, −0·038, −0·643 and 0·095, respectively. These beta values indicate that F and C_r are the main controls of f_m. Neither h/D_r nor D/D_r appears in the equation because their contributions to the explained variance in f_m are not significantly different from 0. In the case of D/D_r, this may be due in part to D being a constant in the data set (i.e. the series 2 experiments).

The exponents on F, R_r, C_r and C in Equation 25 are not significantly different from −4, −0·1, −1 and 0·33, respectively. Consequently, the exponents in Equation 25 are changed to these values, and the intercept is recomputed using non-linear regression analysis. The functional relation then becomes

$$f_m = 0{\cdot}2 F^{-4} R_r^{-0{\cdot}1} C_r^{-0{\cdot}1} C^{0{\cdot}33} \tag{26}$$

with $R^2 = 0{\cdot}95$ and $SEE = 0{\cdot}078$ log units. Figure 3 shows that Equation 26 is a good unbiased estimator of f_m. Equation 26 may now be combined with Equations 12, 15 and 21 to predict f for sediment-laden overland flows on rough mobile beds.

Given that F and C_r are the main controls of f_m, an attempt was made to develop an equation for predicting f_m that contains just F and C_r as the predictive variables. Accordingly, the exponents of −4 and −1 on F and C_r in Equation 26 are retained, and the intercept is recomputed using non-linear regression. The derived equation is:

$$f_m = 0{\cdot}025 F^{-4} C_r^{-1} \qquad C > 0 \tag{27}$$

with $R^2 = 0{\cdot}95$ and $SEE = 0{\cdot}076$ log units (Figure 4). Given (1) that the values of R^2 and SEE for Equations 26 and 27 are almost identical, (2) that the differences between Figures 3 and 4 are very small, and (3) that Equation 27 is simpler than Equation 26, f_m is calculated using Equation 27 rather than Equation 26. Total flow resistance is therefore obtained by summing Equations 11, 15, 21 and 27 which represent f_s, f_f, f_w and f_m, respectively.

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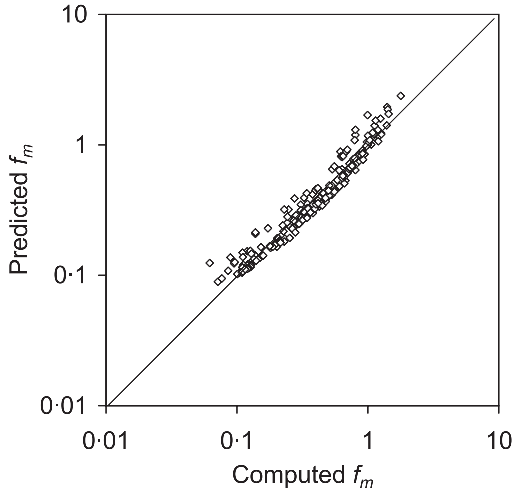

Figure 3. Comparison of computed f_m with f_m predicted by Equation 26.

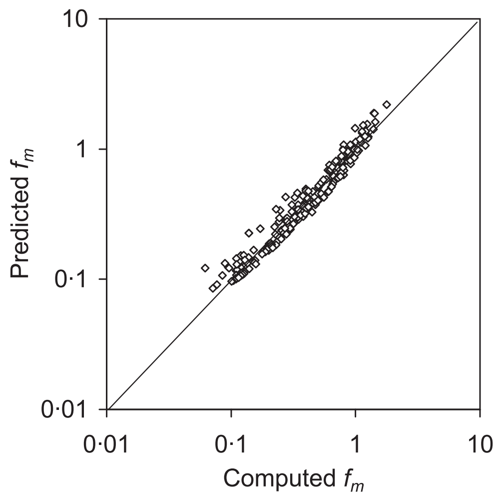

Figure 4. Comparison of computed f_m with f_m predicted by Equation 27.

Predicting Resistance to Overland Flow

Wave and bed-mobility resistance combined

Given the similar structures of the equations for f_w and f_m (i.e. Equations 21 and 27), we investigated the possibility of simplifying the predictive equation for total resistance by combining f_w and f_m (calculated using Equations 21 and 27) into a composite resistance component f_{w+m}. The calculated values of f_{w+m} for the second series of experiments are listed in Appendix 2. The ratio f_{w+m}/f ranges from 0·511 to 0·817 and averages 0·694 (Table II). As the average value of f_m/f on rough mobile beds is 0·322 (Table II), the mean value of f_w/f is 0·372 (= 0·694 − 0·322), indicating that on average f_w is 54 per cent and f_m is 46 per cent of f_{m+w}.

A predictive equation for f_{w+m} was derived by dimensional analysis and multiple regression analysis following the same procedure as was used to develop equations for f_w and f_m. The derived equation for f_{w+m} is:

$$f_{w+m} = 0{\cdot}63F^{-2} \tag{28}$$

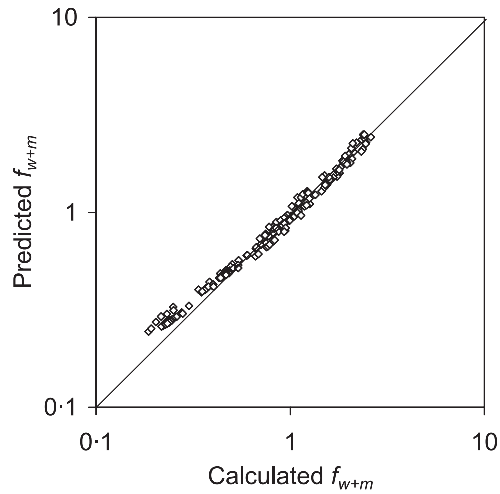

Figure 5. Comparison of computed f_{w+m} with f_{w+m} predicted by Equation 28.

with $R^2 = 0{\cdot}98$ and $SEE = 0{\cdot}042$ log units. Figure 5 indicates that despite its simplicity, Equation 28 is a good unbiased estimator of f_{w+m}. Note that Equation 28 has only one predictive variable, namely F, whereas Equations 21 and 27 each have two, namely F and C_r. Evidently, the contributions of C_r to f_w and f_m cancel out when f_w and f_m are summed before being regressed against f.

Additive nature of the resistance components

That f_w and f_m are additive rather than multiplicative components of flow resistance is demonstrated by performing non-linear regression analyses in which f_w and f_m are each regressed against F^{-2} to determine the value of the intercepts. The resulting equations are:

$$f_w = 0{\cdot}33F^{-2} \tag{29}$$

with $R^2 = 0{\cdot}81$ and

$$f_m = 0{\cdot}28F^{-2} \tag{30}$$

with $R^2 = 0{\cdot}79$. Summing Equations 29 and 30 gives:

$$f_{w+m} = f_w + f_m = 0{\cdot}61F^{-2} \tag{31}$$

which is very close to Equation 28.

Although Equation 28 is the result of a fairly lengthy statistical analysis, the final equation is wholly consistent with the known relation between f and F:

$$f = 8gSh/u^2 = 8SF^{-2} \tag{32}$$

Given that the average value of f_{w+m}/f is 0·694 (Table II) and $S = 0{\cdot}114$, it can be seen that

$$\begin{aligned} f_{w+m} &= 0{\cdot}694f = 0{\cdot}694 \times 8SF^{-2} \\ &= 0{\cdot}694 \times 8 \times 0{\cdot}114 \times F^{-2} \\ &= 0{\cdot}632F^{-2} \end{aligned} \tag{33}$$

which is Equation 28. This finding lends confidence to the methods employed in this study in general and to the dimensional analysis and regression analysis that give rise to Equation 28 in particular.

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Predictive equation for f

Replacing $f_w + f_m$ (that is Equation 21 plus Equation 27) with f_{w+m} (Equation 28), the total resistance f may be predicted by summing Equations 11, 15 and 28:

$$f = \frac{8\tau_s}{\rho u_s^2} + \frac{16}{\pi} C_d \frac{h}{D_r} C_r + 0{\cdot}63F^{-2} \tag{34}$$

Figure 6 confirms that Equation 34 gives good unbiased predictions of total resistance to overland flow on rough mobile beds. However, Equation 34 applies only to the range of conditions represented by the experimental data (Table I). Predictions based on data from outside this range are likely to be in error. (For example, Equation 34 is based on data from flume experiments in which $S = 0{\cdot}114$. If Equation 34 were used to predict f for any other flow with a different slope, the prediction would inevitably be wrong.) Although Equation 34 is not a general equation for predicting f, it demonstrates that resistance to overland flow on rough mobile beds can be well predicted by partitioning the flow resistance into components, developing separate equations for each component, and summing the components.

Conclusion

This study is concerned with overland flows transporting bed load over mobile beds covered with large-scale roughness elements. In the absence of rainfall, the total flow resistance is divided into four components: surface resistance f_s, form resistance f_f, wave resistance f_w, and mobile-bed resistance f_m. Equations for predicting f_s and f_f are borrowed from the literature, while equations for estimating f_w and f_m are developed from two series of flume experiments using dimensional analysis to identify the relevant predictive variables and regression analysis to determine their coefficients. The first series is conducted on smooth fixed beds, and the second on rough mobile beds. The major controls of f_w and f_m are the Froude number F and the concentration of the roughness elements C_r. However, when the equations for f_w and f_m are summed, the C_r terms cancel out, making $f_w + f_m$ an inverse function of F^2. As slope is constant throughout this study, it is not possible to evaluate the nature and importance of S as a control of $f_w + f_m$.

Analyses of the series 1 and 2 experiments enable the contributions of f_s, f_f, f_w and f_m to f to be computed (Table II). An analysis of the first series reveals that in clear-water flows over fixed beds f_w constitutes 52 per cent of f. A similar analysis of the second series indicates that in sediment-laden flows over mobile beds f_w constitutes 37·2 per cent and f_m 32·2 per cent of f. These percentages are consistent with Equations 29 and 30, suggesting that on average $f_w/f_m = 1{\cdot}18$ and that f_w and f_m are both inversely related to F^2.

Given the limited range of hydraulic, sediment and bed roughness conditions represented by the experimental data (for example, S is always 0·114), it is clear that Equation 34 is not a general equation for predicting f. Consequently,

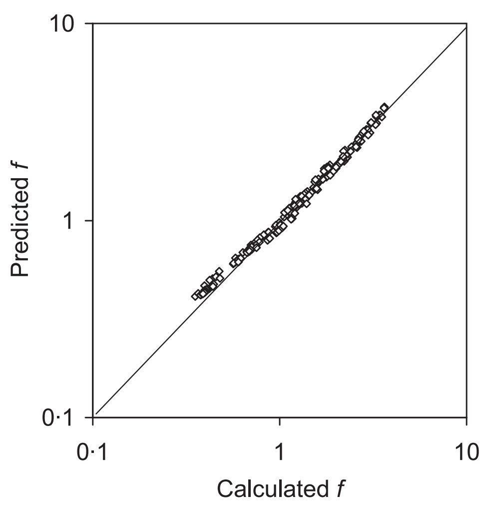

Figure 6. Comparison of computed f with f predicted by Equation 34.

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it should not be used for this purpose outside the range of conditions for which it was developed (i.e. Table I). However, the methodology employed here can be extended to include other experimental data representing different conditions. Should such data become available, the present methodology could be used to develop a general equation for predicting total *f*.

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